# III EMALCA COSTA RICA

# Sede Regional del Atlántico

# UNIVERSIDAD DE COSTA RICA

## Report 2014

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## 1. Introduction

From the 10th to the 21th of February 2014, an EMALCA was held at the site of the Sede Regional del Atlántico of the Universidad de Costa Rica in Turrialba, Cartago, Costa Rica. It was attended by students from Nicaragua, El Salvador, Honduras, Guatemala, Panamá and Costa Rica. The main topics of the school were combinatorics, geometry, numerical methods, graph theory and topology. There was a great diversity in the background of the participating students and this was a good match for the different requirements of the courses given at the school.

## 2. Administrative report

The coordinators of the event were Pedro Méndez and José Alexander Ramírez from the Universidad de Costa Rica and Rafael Labarca from Universidad de Santiago de Chile. The scientific committee consisted of the professors Dr. Rafael Labarca; Dr. Pedro Méndez and Dr. José Seade from the IMAT at UNAM, Cuernavaca, México.

The local organizing committee was comprised of the following people: Carlos Ulate from the Sede Regional de Occidente of the Universidad de Costa Rica; José Rosales from the Instituto Tecnológico de Costa Rica; Pedro Méndez and José Alexander Ramírez from the Universidad de Costa Rica and Ana Catalina Camacho from the Sede Regional del Atlántico of the UCR. José A. Ramírez was the coordinator of the organizing committee.

The courses took place a special conference room and in the auditorium and the talks were held at the same place. The location of the activities was changed from the first to the second week. Refreshments were served nearby. At a given time, all the rooms were within a 50m radius.

Participants (students and professors) were given breakfast, lunch, dinner and mid-morning and mid-afternoon refreshments. A catering service at the university cafeteria was hired for these purposes.

The students stayed at the university housing of the Sede Regional. These are shared rooms with several beds per room. There were communal tables for studying and a social area with kitchen and other equipment. Three sets of shared bathrooms were available for students use.

Costa Rica's and international students were offered transportation from the central campus of the UCR to the Turrialba campus at the beginning and ending of each week. A few international students were picked up and dropped of at the bus station or airport, but most of them arrived directly to the central campus.

The professors stayed at a hotel in Turrialba called "Hotel Wagelia". They were given individual rooms that had breakfast included.

The school comprised four courses. Every course one of them included six class sessions of 90 minutes each and an evaluation. The class sessions were held from Monday to Thursday and the exam was applied on Friday. Class notes were given for each course.

Also, four two-part talks were given. Each talk was three hours long but was delivered in two parts of 90 minutes each.

A web site with basic information was created at emalca.emate.ucr.ac.cr

The attendance to the school was different on each week. Because of personal commitments, some costarican students were unable to attend both weeks. Eighteen costarican students attended the first week. Their names are listed in the table below

| # | Name | ID number | University |
|---|---|---|---|
| 1 | Bryan Andrés Gómez Vargas | 1-1371-0405 | UCR San Ramón |

| | | | |
|---|---|---|---|
| 2 | Bryan Rodríguez Castro | 1-1395-0025 | ITCR |
| 3 | Carolina Montoya Rivera | 3-0481-0225 | UCR San Pedro |
| 4 | Catalina Camacho Navarro | 3-0429-0652 | UCR Turrialba |
| 5 | Cristhian Julián Peña Sierra | 1-17000932408 | UCR San Pedro |
| 6 | Daniel Alberto Aguilar Álvarez | 1-1426-0060 | UCR San Pedro |
| 7 | Jesús Rodríguez Rodríguez | 2-0588-0957 | UCR San Ramón |
| 8 | Jorge Asdrúbal Villalobos Alvarado | 2-0712-0981 | UCR San Pedro |
| 9 | Jorge Luis Salazar Chaves | 2-0637-0590 | UCR San Ramón |
| 10 | José Emmanuel Chacón Chavarría | 1-1517-0112 | UCR San Pedro |
| 11 | José Pablo Flores Zúñiga | 4-0194-0125 | UNED |
| 12 | Luis Alfredo Valerín Lara | 7-0207-0928 | UCR San Pedro |
| 13 | Luis Enrique Amador Araya | 1-1474-0840 | UCR San Pedro |
| 14 | Luis Gómez Rodríguez | 1-1139-0488 | UCR San Pedro |
| 15 | Mario Molina Gómez | 3-0450-0979 | ITCR |
| 16 | Norman Noguera Salgado | 1-1142-0631 | UCR San Ramón |
| 17 | Priscilla Calderón Jiménez | 3-0456-0120 | ITCR |
| 18 | Vinicio Vega Soto | 3-0435-0730 | ITCR |

Eighteen costarican students attended the second week

| # | Name | ID number | University |
|---|---|---|---|
| 1 | Anthony Santiago Chaves | 1-1556-0823 | UCR San Pedro |
| 2 | Bryan Andrés Gómez Vargas | 1-1371-0405 | UCR San Ramón |
| 3 | Bryan Rodríguez Castro | 1-1395-0025 | ITCR |
| 4 | Carolina Montoya Rivera | 3-0481-0225 | UCR San Pedro |
| 5 | Catalina Camacho Navarro | 3-0429-0652 | UCR Turrialba |
| 6 | Cristhian Julián Peña Sierra | 1-17000932408 | UCR San Pedro |
| 7 | Daniel Alberto Aguilar Álvarez | 1-1426-0060 | UCR San Pedro |
| 8 | Hugo Alberto Flores Arguedas | 1-1059-0488 | UCR San Pedro |

| | | | |
|---|---|---|---|
| 9 | Jesús Rodríguez Rodríguez | 2-0588-0957 | UCR San Ramón |
| 10 | Jorge Asdrúbal Villalobos Alvarado | 2-0712-0981 | UCR San Pedro |
| 11 | Jorge Luis Salazar Chaves | 2-0637-0590 | UCR San Ramón |
| 12 | José Emmanuel Chacón Chavarría | 1-1517-0112 | UCR San Pedro |
| 13 | José Pablo Flores Zúñiga | 4-0194-0125 | UNED |
| 14 | Luis Alfredo Valerín Lara | 7-0207-0928 | UCR San Pedro |
| 15 | Luis Enrique Amador Araya | 1-1474-0840 | UCR San Pedro |
| 16 | Luis Gómez Rodríguez | 1-1139-0488 | UCR San Pedro |
| 17 | Mario Molina Gómez | 3-0450-0979 | ITCR |
| 18 | Priscilla Calderón Jiménez | 3-0456-0120 | ITCR |
| 19 | Vinicio Vega Soto | 3-0435-0730 | ITCR |

Nineteen foreign students attended both weeks

| # | Name | Passport number | Country | University |
|---|---|---|---|---|
| 1 | Ledy Janette Quezada Martínez | C01498192 | Nicaragua | UNAN-León |
| 2 | Lissette Quintero Vargas | C01617298 | Nicaragua | UNAN-León |
| 3 | Rommel Guido Carcache | C01409469 | Nicaragua | UNAN-León |
| 4 | Melissa Lizbeth Velásquez Castillo | C01608645 | Nicaragua | UNAN-Managua |
| 5 | Melvin Alexander Rovelo Quintanilla | Z014783 | Honduras | UNAH |
| 6 | Ángela Paola Izaguirre Bonilla | C984812 | Honduras | UNAH |
| 7 | Francisco José Martínez Figueroa | 222663650 | Guatemala | Del Valle |
| 8 | Fernando José Mazariegos Camas | 15950461K | Guatemala | San Carlos |
| 9 | Mónica Lucía Cabria Zambrano | 242294642 | Guatemala | San Carlos |
| 10 | María Emilia Calderón Font | 211495186 | Guatemala | Del Valle |
| 11 | Ronald Oliverio Chubay Gallina | 222429321 | Guatemala | San Carlos |
| 12 | Mario Roberto Gómez Flores | 133324150 | Guatemala | Del Valle |
| 13 | José Jonathan Sánchez Vásquez | A04335997 | El Salvador | UES-San Miguel |

14 Willian Armando Miranda Tobar B04291196 El Salvador UES-San Salvador

15 José Alfredo Hernández Pérez A04562916 El Salvador UES-San Miguel

16 Manuel Bernardo Trejo Montiel A04261573 El Salvador UES-San Miguel

17 José David Zaldívar Olivares A04426691 El Salvador UES-San Miguel

18 Julieta Elisa Quiroz Pérez PA0005272 Panamá UNACHI

19 Ted Augusto Jiménez Tejeira PA0005753 Panamá UNACHI

3. Financial report

The contributions to the school can be summarized as follows:

1. Students stayed at the UCR university residences at no cost.

2. Universidad de Costa Rica, contributed with the meals for its students and coffee break refreshments for all students.

3. ITCR paid for the meals of its students

4. Accommodation for professors was covered by UCR

5. Other expenses (photocopying, diplomas, etc.) were covered by the UCR

6. CIMPA is expected to pay for stipends and meals of foreign students

7.- Airfare and partial stipends for professors Eugenia O'Reilly and José Seade were covered by the Universidad Autonóma de México from México.

8.- Airfare and partial stipends for professors Maya Stein and Rodolfo Rodríguez were covered by the Centro de Modelamiento Matemático of the Universidad de Chile from Chile

9.- Airfare and partial stipends for professor Yuriko Baldin were covered by the Instituto Nacional de Ciencia e Tecnología de Matemática from Brazil.

10.- Airfare and partial stipends for professors Renzo Cavalieri and James Wilson were covered by the Colorado State University from USA.

The total number of foreign participants in EMALCA was nineteen. We had 63 applications of which we sent acceptances to 25. Some had to cancel at the last minute.

The CIMPA money was budgeted for stipends and meals.

The stipends were calculated on the basis of country of origin. We quoted a standard two way ticket from the capital of the foreign country to San José, Costa Rica. To that we added the cost of a visa for a national of that country (only Nicaragua has that requirement). Finally we gave them 20 dollars to cover the meals of the weekend in Turrialba (15th and 16th of February).

4. Scientific report

The four courses were:

1. Introduction to Topology. Prof. José Seade, Universidad Autónoma de Mexico, Mexico

Topological Spaces, Topology in Euclidean space, topology in metric spaces, Definition and examples of topological spaces, Topology subspaces, bases and sub-bases, product topology, Closed sets and accumulation points, compact sets, connected sets, functions between topological spaces, Continuous Functions, Quotient Spaces, Actions of groups, Examples: Cylinders and tori, Submersiones , injections and homeomorphisms, Homotopia, Retractions and deformations, Topological spaces with additional structure, topological manifolds, knots in space, vector bundles, How to distinguish topological spaces, The fundamental group, covering Projections, Examples

2. Introduction to Graph Theory. Prof. Dr. Maya Stein. CMM , University of Chile. Chile.

Introduction, coloring of edges and vertices, List coloring, Brooks Theorem, Vizing theorem, Konig Theorem, four color theorem, List coloring Conjecture, Galvin Theorem, Ramsey theory, Ramsey's theorem, upper and lower bounds, applications, Schur theorem, Extremal Graphs theory, Dirac's theorem, Hall Theorem, Mantel theorem, Turan Theorem, Erdos-Stone-Simonovits Theorem, Erdos-Sos Conjecture, Hadwiger Conjecture, regularity, probabilistic methods, ` Random Dependent Choice '.

3. An Introduction to Combinatorics. Prof. Dr. Eugenia O'Reilly Universidad Autónoma de Mexico, Mexico

Combinations, permutations pigeonhole principle, relationships, equivalence and partitions. Introduction to the theory of graphs, Block layouts and structures of incidence.

4. Numerical Finite Element Solution of Partial Differential Equations. Prof. Dr. Rodolfo Rodriguez. Universidad de Concepción. Chile.

Weak formulation of elliptic differential equations. Sobolev spaces . Lax- Milgram lemma. Galerkin method. Finite Element Method. Cea’s lemma. Error estimate. Computational implementation of the finite element method for elliptic problems. Assemble. Calculating elementary matrices. Imposition of boundary conditions. Solving systems of linear equations arising. The heat equation. Finite element solution of parabolic problems. Explicit and implicit schemes. Wave equation. Finite element solution of linear hyperbolic problems. Explicit and implicit schemes. Free vibration modes. Finite Element Solution of spectral problems. Generalized eigenvalue problems.

The four (two part) talks were:

5. Prof. Dr. Renzo Cavalieri, Colorado State University. United States of America.
Enumerative Geometry and Modular Spaces.

Summary: Enumerative geometry studies questions like How many geometric objects of a given type satisfy a given number of geometric constraints? for example 1. How many lines in the plane passing through two distinct points ?

2 . How many lines in the tri -dimensional space are incident to four given lines ?

While the answer to question ( 1) is known since high school, the question ( 2) , the answer is two illustrates the complexity of these problems can result.

The modern approach to study these problems is to transform them into questions of intersection theory of moduli spaces . A modular space is a bijective geometric objects we wish to study space and geometrical structure which reflects the geometry of the objects to be configured. Intuitively move continuously along the moduli space corresponds to geometrically distort its object. By imposing geometric constraints subspaces of the moduli space defined thus fulfilling several simultaneous constraints intersect corresponds to modular spaces.

In these lectures we will use the enumerative geometry to enter the fascinating world of Modular Spaces . Study some concrete examples such as projective spaces , and spaces punctuated Grassmanianos rational curves , and give indications as to study its geometry. As applications of this theory will get answers to some questions of enumerative geometry. Finally , we will present an overview of how some questions of high difficulty , such as counting the number of rational plane curves of degree d passing through 3d- 1 distinct points have been resolved.

6. Prof. Dr. James B. Wilson , Colorado State University. United States of America. Neoclassical geometry

Summary:The classical geometry is in part the study of the orthogonality relations between vectors in a vector space . Therefore domestic products are routine to relate algebraic objects with geometric problems tools. The introduction of this geometric connection was very useful in classifying and identifying possible geometries geometries irreducible algebra through its behavior. Mixing the algebraic and geometric reasoning now understands many significant properties of classical geometry.

Neoclassical Geometry studies the more complex problem arises when imposing multiple geometries in a vector space , for example if it is necessary that two vectors are orthogonal with respect to an inner product over . This reflects the common reality that usually there are multiple applications geometric constraints in the same space that are not related in an obvious way .

In the last half- century, several results has improved algebra our understanding of the classical neo geometry using powerful results from the theory ring . Through these connections you can now identify each neo classical geometry a string of smaller classical geometries that are unique to this structure. This behavior has been implemented in computer algorithms that are able to identify neo classical geometries in dimension 100 minutes.

7. Trigonometry in finite planes. Prof. Dr. Joseph C. Varilly . University of Costa Rica . Costa Rica .

A finite plane is a square board where each square represents a point. A line is a periodic array of cells, but one cannot talk of segments for lack of a relationship of linear order. However, it is possible to define and establish certain trigonometric relationships by quadratic functions of coordinates in a finite body. Thus, the laws of trigonometry take on new life in finite planes.

8. The concept and applications of curvature. Prof. Dr. Yuriko Baldin, Federal University of Sao Carlos. Brazil. (joint work with Ferninado Ararello, University of Torino).

Abstract: In this talk we recall an intuitive justification of the concept of curvature of a two-dimensional surface and discuss the progress of mathematics in the last century as the concept was formalized and generalized, for example with Ricci curvature and solutions of old conjectures. Still leaving with intuitive motivation, we discuss the concept of curvature through parallel transport of vectors and show how this relates to applications in modern, as the effect of Pancharatnam-Berry phase in optical systems. This talk is based on work conducted as part of The Klein Project for the 21th century the ICMI-IMU.

The four courses had evaluation and this took place on Friday each week. The reported grades are shown in the following table.

| Student's name | Combinatorics | Topology | Graph theory | Numerical methods |
|---|---|---|---|---|
| Ángela Paola Izaguirre Bonilla | | 6 | 7 | |
| Anthony Santiago Chaves | | | 10 | |
| Bryan Andrés Gómez Vargas | | 10 | 10 | |
| Bryan Rodríguez Castro | 6 | 8 | | |
| Carolina Montoya Rivera | | 7 | A | |
| Catalina Camacho | 9 | A++ | | |
| Cristhian Julián Peña Sierra | 6 | | 9 | |
| Daniel Alberto Aguilar Álvarez | | 9 | 9 | |
| Fernando José Mazariegos Camas | | 10 | A+ | |
| Francisco José Martínez Figueroa | | 10+ | A+ | |
| Hugo Alberto Flores Arguedas | | | 9 | |
| Jesús Rodríguez | 10 | 10 | | |
| Jorge Asdrúbal Villalobos Alvarado | | 8 | 9 | |
| Jorge Luis Salazar Chaves | | 10 | 10 | |
| José Alfredo Hernández Pérez | | 9 | 8 | |
| José David Zaldívar Olivares | | 6 | A | |

| | | | | | |
|---|---|---|---|---|---|
| José Emmanuel Chacón Chavarría | | | 10 | A+ | |
| José Jonathan Sánchez Vásquez | | 7 | | 7 | |
| José Pablo Flores Zúñiga | | NA | | 8 | |
| Julieta Elisa Quiroz Pérez | | NA | | 8 | |
| Ledy Janette Quezada Martínez | 3 | | | 8 | |
| Lissette Quintero | 2.5 | | 8 | | |
| Luis Alfredo Valerín Lara | | 9 | | 9 | |
| Luis Enrique Amador Araya | | 9 | | 9 | |
| Luis Gómez | 8 | A+ | | | |
| Manuel Bernardo Trejo Montiel | | | 7 | | 9 |
| María Emilia Calderón Font | | 8 | A+ | | |
| Mario Molina Gómez | 9 | A | | | |
| Mario Roberto Gómez Flores | | 8 | A+ | | |
| Melissa Lizbeth Velásquez Castillo | | | 7 | | 8 |
| Melvin Alexander Rovelo Quintanilla | | | 7 | A | |
| Mónica Lucía Cabria Zambrano | 8 | | NA | | |
| Norman Noguera Salgado | | 10 | | | |
| Priscilla Calderón Jiménez | 5 | | NA | | |
| Rommel Guido Carcache | | NA | NA | | |
| Ronald Oliverio Chubay Gallina | | 7 | | 7 | |
| Ted Augusto Jiménez Tejeira | 5 | | | 5 | |
| Vinicio Vega Soto | 5 | NA | | | |
| Willian Armando Miranda Tobar | | | 9 | | 10 |

The scales were developed individually by each professor. The table is a transcription of the professor's report. The courses on combinatorics and topology have a passing grade of 6. The course on numerical methods had a passing grade of 7. A means "passed" ("Aprueba") and NA means "not passed" ("No Aprueba"). A plus sign ("+") indicates that the student's test was particularly good.